Intelligent Optimization for an Integrated Inventory Model with Multi-Product and Multi-Storehouse

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***Abstract*—Multi-product and multi-storehouse retailer companies are very common all over the world. For such companies, inventory management is a critical activity for determining their success. In this context, mathematical inventory models are useful to help finding optimal solutions for answering typical retailer company questions such as: when to order, how many products to order, where to store and what is the optimal retail price for a given product. In this paper, we propose a novel integrated supplier-buyer inventory model able to optimize operations of multi-product and multi-storehouse retailers. The optimal solution to the inventory model is found using Swarm Intelligence based on Particle Swarm Optimization (PSO). Experimental results of the proposed model showed that the considered optimization is feasible and is not excessively computationally intensive. In addition to that, our approach allows the inclusion of strategic policies in that optimization model.**

Index Terms— **Integrated Inventory Model, Particle Swarm Optimization, Multi-Storehouse, Multi-Product.**

I. INTRODUCTION

Optimization of inventories using models with emphasis in multi-product and multi-storehouse is completely new but practical contributions are still rare. Conversely, relevant contributions are available for single-product and single-storehouse. This is due, mostly, to the trade-off between the optimization techniques and the complexity involved in the analytical manipulations involved in inventory models that tackle multi-product and multi-storehouse. So, purchasing managers who deal with distributed stocks comprised of many products, are often equipped with systems that are not fully capable of realizing the necessary optimization. Supermarkets, for example, retail at about 10000 products in each of their storehouses [1]. So, it is easy to imagine how difficult the purchasing manager's job is, and how important it would be for this person to be appropriately helped of.

In addition to the decision questions mentioned above, a purchasing manager must choose the appropriate supplier of a product or a group of products in stock. Many studies have addressed this subject and tried to reduce the complexity of the managerial decision [2]–[3], by focusing only on the buyer's needs. Alternatively, other researchers have proposed a coordinated inventory management throughout the supply chain [4]–[7]. In this case, the main objectives are to avoid the complexities involved in the decision, to reduce costs related to inventory purchase and to improve the appropriateness to customer requirements response. In light of so many benefits, growing synchronization among suppliers, customers and third-party providers is a marked tendency [8].

In line with the idea of cooperation throughout the supply chain, Abad and Jaggis [9] developed an integrated supplier-buyer inventory model that allows the supplier to offer to the buyer a delay in payment for goods already delivered. That delay consists of a "one-part" trade credit, *i.e.* the buyer has only one alternative to postpone the payment after the delivery. For example, by the trade credit denoted as "net 60", the buyer can postpone the full payment for 60 days after the delivery. Nevertheless, without encouragement, the buyer will not make early payments. Ho *et al.* [10] then extended Abad and Jaggis' model [9] to an integrated supplier-buyer inventory model with a "two-part" trade credit policy. As an example of this new policy, a trade credit term denoted as "2/20 net 90" offers the buyer a two percent discount over the total purchasing cost if the payment is made in 20 days. Otherwise, the buyer can postpone the full payment up to 90 days after the delivery.

In this paper, a new inventory model is put forward by extending the model proposed by Ho *et al.* [10]. Our approach is able to tackle multi-product and multi-storehouse. In the new model, search and suggestions of optimal policies are based on Particle Swarm Optimization (PSO). In addition to that, we devised a goal reaching mechanism capable of receiving targets to be met according to buyer's and/or supplier's strategic policies (*e.g.* minimal retail price for a product and buyer's minimal margin profit).

Some experimental results are presented to illustrate the flexibility of our approach of using PSO in inventory model.

II. AXIOMS AND MATHEMATICAL NOTATION

Before presenting the new mathematical inventory model, it is necessary to establish a set of axioms and to introduce the mathematical notation used to define the new model. Notice that axioms and notation is adapted and extended from Ho *et al.* [10]:

1) A single buyer retails products from only one supplier;
2) Each product $i \in [1,2,\dots,q]$ is sold in all of the buyer's storehouses $k \in [1,2,\dots,b]$;
3) The i[th] product demand rate is a function of the retail price in the k[th] storehouse as follows: $D_{ik}(p_{ik}) = a_{ik}\, p_{ik}^{-\delta_{ik}}$. The constant $a_{ik} > 0$ is a scaling factor and $\delta_{ik} > 0$ is a price-elasticity coefficient;

4) T_i is the buyer's inventory replenishment time for the i[th] product. Then, $Q_i = \sum_{k=1}^{b} D_{ik} T_i$ is the order quantity for the i[th] product;
5) The cost for the buyer to order Q_i units of the i[th] product is S_{B_i};
6) The supplier produces $n_i Q_i$ units (where n_i is an integer) units of the i[th] product and makes periodical deliveries to the buyer for each product. Each delivery contains Q_i units of the i[th] product. The buyer stores them in a distribution center and reassures that those products will be on time in the storehouses.
7) For simplicity, questions related to the product transportation logistic process were avoided;
8) The supplier's setup cost to produce in batches of size $n_i Q_i$ is S_{V_i};
9) Shortages are not allowed;
10) To estimate a production rate R_i to avoid shortages and to calculate the supplier's carrying cost, the capacity utilization is defined as $\rho_i = D_i / R_i$, where $D_i = \sum_{k=1}^{b} D_{ik}$ and $\rho_i < 1$;
11) Both the supplier and the buyer pay opportunity and carrying costs. r_{V_i} and r_{B_i} are the supplier's and the buyer's carrying cost rates, respectively, for the i[th] product;
12) In offering a delay time for the buyer to pay for goods already delivered, the supplier endures a capital opportunity cost at rate I_{Vp_i} for the i[th] product;
13) Once the buyer pays for the ordered goods, it incurs a capital opportunity cost for the buyer ate rate I_{Bp_i};
14) After the delivery, if the buyer pays in M_{1_i} days, the supplier will offer a discount $0 < \beta_i < 1$ over the total purchasing price for the i[th] product. Otherwise, the buyer will pay the total purchasing price in M_{2_i} days, where $M_{2_i} > M_{1_i} \geq 0$;
15) The unitary production cost for the supplier, in dollar, is c_i, for the i[th] product. The buyer's purchasing and retail price, for that product, are $\$ v_i$ and $\$ p_{ik}$, respectively. Where p_{ik} is the retail price of the i[th] product in the k[th] storehouse and $p_{ik} > v_i > (1-\beta_i) v_i > c_i$;
16) If the buyer makes an early payment, the supplier's advantage for that will be quantified as a cash flexibility rate f_{Vc_i} for the i[th] product, during the period $\lfloor M_{1_i}, M_{2_i} \rfloor$;
17) I_{Be_i} is the buyer's interest rate earned during the credit period (i.e. M_{1_i} or M_{2_i}) for the i[th] product;
18) $\vec{n} = (n_1, n_2, \dots, n_q)$, $\vec{T} = (T_1, T_2, \dots, T_q)$, $\vec{p}_i = (p_{i1}, p_{i2}, \dots, p_{ib})$ and $\vec{p} = (\vec{p}_1, \vec{p}_2, \dots, \vec{p}_b)$.

III. Mathematical Inventory Model Formulation

The objective of this section is present how to model the joint expected total profit (*i.e.* the sum between the supplier's and buyer's profit) per unit of time. Firstly, we formulate the total supplier's expected profit and then the buyer's total expected profit. For both the supplier and buyer we present first the costs and, subsequently, the income and then the profit.

The costs associated to the entire supplier's manufacture process, for each i[th] product, are: (i) the production cost: $c_i D_i$; (ii) the batch set-up cost, *i.e.* the cost to manufacture the products in batches of size $n_i Q_i$: $S_{V_i} / n_i T_i$, where $n_i T_i$ is the duration of the entire buyer's retail cycle; (iii) the carrying cost, using the Joglekar's approach [11], given by (1):

$$\left(c_i r_{V_i} + c_i I_{Vp_i}\right)\left(D_i T_i / 2\right)\left[(n_i - 1)(1-\rho_i) + \rho_i\right] \Rightarrow D_i T_i c_i \left(r_{V_i} + I_{Vp_i}\right)\left[(n_i - 1)(1-\rho_i) + \rho_i\right]/2; \quad (1)$$

(iv) the opportunity cost, given by (2):

$$(1 - g_j \beta_i) v_i I_{Vp_i} D_i M_{j_i},\ j = 1,2, \text{ and } g_1 = 1, g_2 = 2 \quad (2)$$

The supplier's gain is obtained from: (i) capital sales revenue: $(1 - g_j \beta_i) v_i D_i$; (ii) the possible advantage from early payment: $g_j (1-\beta_i) v_i f_{Vc_i} D_i \left(M_{2_i} - M_{1_i}\right)$.

Consequently, the total expected supplier profit per unit time is given by (3):

$$TVP_j\left(\vec{n}, \vec{p}, \vec{T}\right) = \sum_{i=1}^{q} VP_j(n_i, \vec{p}_i, T_i) \Rightarrow \sum_{i=1}^{q} \left\{ \begin{array}{l} \left(\left(1 - g_j \beta_i\right) v_i D_i\right) + g_j (1-\beta_i) v_i f_{Vc_i} D_i \left(M_{2_i} - M_{1_i}\right) \\ -\left(c_i D_i\right) - \left(S_{V_i} / n_i T_i\right) \\ -\left(D_i T_i c_i \left(r_{V_i} + I_{Vp_i}\right)\left[(n_i - 1)(1-\rho_i) + \rho_i\right]\right)/2 \\ -(1 - g_j \beta_i) v_i I_{Vp_i} D_i M_{j_i} \end{array} \right\} \quad (3)$$

$$j = 1,2 \text{ and } g_1 = 1, g_2 = 0.$$

On the other hand, the buyer endures costs, for each i[th] product, from: (i) the total purchasing cost: $(1 - g_j \beta_i) v_i D_i$; (ii) the cost per order: S_{B_i} / T_i; (iii) the carrying cost, considering an average inventory of $Q_{ik}/2$ over the retail cycle, given by (4):

$$\sum_{k=1}^{b}\frac{(1-g_j\beta_i)v_i r_{B_i}Q_{ik}}{2}\Rightarrow\frac{(1-g_j\beta_i)v_i r_{B_i}T_i}{2}\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}} \Rightarrow ((1-g_j\beta_i)v_i r_{B_i}T_iD_i)/2; \tag{4}$$

(iv) the opportunity cost (if $T_i \geq M_{j_i}$, i.e. the buyer must pay off before the end of the retail cycle. If $T_i < M_{j_i}$, the buyer does not pay opportunity cost), given by (5):

$$\frac{(1-g_j\beta_i)v_i I_{Bp_i}}{T_i}\sum_{k=1}^{b}\left\{\int_{M_{j_i}}^{T_i}D_{ik}(t-M_{j_i})dt\right\}\Rightarrow \frac{(1-g_j\beta_i)v_i I_{Bp_i}(T_i-M_{j_i})^2D_i}{2T_i}. \tag{5}$$

When calculating the buyer's gains it is necessary to distinguish two possible situations: (i) if $T_i < M_{j_i}$ (*i.e.* the buyer retails all units of the i[th] product before the payment date), so the buyer earns interest during all the credit period and the gain is expressed as by (6):

$$\frac{1}{T_i}\left[\sum_{k=1}^{b}\left\{p_{ik}I_{Be_i}\int_0^{T_i}D_{ik}tdt+p_{ik}I_{Be_i}D_{ik}T_i(M_{j_i}-T_i)\right\}\right]\Rightarrow I_{Be_i}(M_{j_i}-T_i/2)\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}; \tag{6}$$

(ii) if $T_i \geq M_{j_i}$ (i.e. at the payment date, the buyer still has items of the i[th] product in stock), so the buyer earn interest only up to the payment date and that income is given by (7):

$$\frac{I_{Be_i}}{T_i}\left[\sum_{k=1}^{b}\left\{p_{ik}\int_0^{T_i}D_{ik}tdt\right\}\right]\Rightarrow\left(I_{Be_i}M_{j_i}^2\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}\right)\Big/2T_i \tag{7}$$

At last, (iii) the total revenue value is $a_{ik}p_{ik}^{-\delta_{ik}+1}$. Therefore, the buyer's total expected profit per unit time is given by (8):

$$TBP_j(\vec{p},\vec{T})=\sum_{i=1}^{q}\begin{cases}BP_{j1}(\vec{p}_i,\vec{T}_i) \text{ if } T < M_{j_i}\\ BP_{j2}(\vec{p}_i,\vec{T}_i) \text{ if } T \geq M_{j_i}\end{cases} \quad j=1,2, \tag{8}$$

where

$$BP_{j1}(\vec{p}_i,\vec{T}_i)=\left\{\begin{array}{l}\left(\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}\right)+\left(I_{Be_i}M_{j_i}^2\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}\right)\Big/2T_i\\ -((1-g_j\beta_i)v_iD_i)-(S_{B_i}/T_i)\\ -((1-g_j\beta_i)v_i r_{B_i}T_iD_i)/2\end{array}\right\}, \tag{9}$$

and

$$BP_{j2}(\vec{p}_i,\vec{T}_i)=\left\{\begin{array}{l}\left(\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}\right)+\left(I_{Be_i}M_{j_i}^2\sum_{k=1}^{b}a_{ik}p_{ik}^{-\delta_{ik}+1}\right)\Big/2T_i\\ -((1-g_j\beta_i)v_iD_i)-((1-g_j\beta_i)v_i r_{B_i}T_iD_i)/2\\ -(S_{B_i}/T_i)-((1-g_j\beta_i)v_iI_{Bp_i}(T_i-M_{j_i})^2D_i)/2T_i\end{array}\right\}. \tag{10}$$

Finally, the total joint expected profit per unit time is given by (11):

$$\Pi_j(\vec{n},\vec{p},\vec{T})=\sum_{i=1}^{q}\begin{cases}\Pi_{j1}(n_i,\vec{p}_i,\vec{T}_i) \text{ if } T < M_{j_i}\\ \Pi_{j2}(n_i,\vec{p}_i,\vec{T}_i) \text{ if } T \geq M_{j_i}\end{cases} \quad j=1,2, \tag{11}$$

where

$$\Pi_{j1}(n_i,\vec{p}_i,\vec{T}_i)=VP_j(n_i,T_i)+BP_{j1}(\vec{p}_i,T_i), \tag{12}$$

and

$$\Pi_{j2}(n_i,\vec{p}_i,\vec{T}_i)=VP_j(n_i,T_i)+BP_{j2}(\vec{p}_i,T_i). \tag{13}$$

IV. Particle Swarm Optimization

Swarm intelligence is a computational intelligence sub-area whose techniques are inspired on an emergent collective and collaborative behavior of a group of animals (*e.g.* ants – Ant Colony Optimization [12], birds – Particle Swarm Optimization [13]).

The Particle PSO technique was developed by Kennedy and Eberhart [13] and is largely applied to determine maximal or minimal points of a non-linear function in a continuous search spaces. To PSO, the swarm of particles represents a set of potential solutions of an optimization problem and each particle's behavior is separately governed by two adapted physical properties, position and velocity, according to equations (14) and (15). Hence, the particles can "fly" throughout the domain function, guided by those equations, toward maximal or minimal points.

$$\vec{v}(t+1)=w\vec{v}(t)+c_1r_1(\vec{P}_l(t)-\vec{x}(t))+c_2r_2(\vec{P}_g(t)-\vec{x}(t)), \tag{14}$$
$$\vec{x}(t+1)=\vec{x}(t)+\vec{v}(t+1). \tag{15}$$

In both equations, r_1 and r_2 are random values uniformly distributed in $[0,\ldots,1]$ used, among other reasons, to avoid local minimal or maximal points. $\vec{P}_l$ is a vector that represents the position of a local maximum or minimum point of a particle refer to its specific search process. $\vec{P}_g$ is a vector that points to a global maximum or minimum point, relative to all particles. Once that global best value is discovered, in the next iteration, *i.e.* $t+1$, that information is shared between all particles. ω is the inertial factor), c_1 is the cognitive coefficient and c_2 is the social coefficient. Together they comprise the parameters to the algorithm and need to be adjusted to every specific optimization problem.

The value of ω represents a direct influence over the convergence time (*i.e.* the greater ω value is, the more t iterations will be necessary for the PSO to achieve convergence). If $c_1 > c_2$, each particle's trajectory will be near to an area along its own $\vec{P}_l$. Then, the resultant search process will be decentralized. In other hand, if $c_1 < c_2$, the particles will explore an area along the local pointed by $\vec{P}_g$. Those search modes are called exploration and exploitation, respectively. For more information about parameter selection and convergence processes of PSO see [14] and [15].

Despite the adapted physical rules of the PSO are relatively simple, the resultant emergent swarm behavior, taking into account the collaboration among the entities (*e.g.* shared information), can be very useful to solve many optimization problems. Because of that relatively simple individual behavior, it's possible to increase the population size in a swarm and still maintain the low computational cost of the search process.

V. Computational Optimization Model

In this section, we adapt the necessity to choose an appropriate policy to that Economic Order Quantity (EOQ) model to an optimization problem and we use a PSO algorithm to solve that problem. In our approach, we assume two possibilities for an appropriate decision: (i) the one that produces the maximal joint expected profit and (ii) the one that best fits targets defined for the buyer and/or for the supplier.

A purchasing manager's decision in the new inventory model is represented as $(\vec{n},\vec{p},\vec{T})$ and Equation (11) is used to evaluate the purchasing function for that decision. Therefore, the resultant optimization problem is to find $(\vec{n},\vec{p},\vec{T})$ that produces the maximal values for $\Pi_j(\vec{n},\vec{p},\vec{T})$.

Once $\Pi_j(\vec{n},\vec{p},\vec{T})$ is obtained from a sum between two other values, there are three undesirable situations when using only this simple approach to qualify a decision: let $(\vec{n}^1,\vec{p}^1,\vec{T}^1)$ and $(\vec{n}^2,\vec{p}^2,\vec{T}^2)$ be two different decisions and $\Pi_j(\vec{n}^1,\vec{p}^1,\vec{T}^1) > \Pi_j(\vec{n}^2,\vec{p}^2,\vec{T}^2)$, then we can get (i) $TVP_j(\vec{n}^1,\vec{p}^1,\vec{T}^1) \le 0$ (*i.e.* the first decision implies loss for the supplier), (ii) $TBP_j(\vec{n}^1,\vec{p}^1,\vec{T}^1) < 0$ (*i.e.* the first decision implies loss for the buyer) and (iii) $p_{ik} \le v_i$ for at least one product (*i.e.* the retail price for some products in some storehouses is less than or equals to its vendor price). That is, despite of the first decision results more profit than the second one, it is not an appropriate decision. Then, to avoid those situations, it was formulated the following restriction to a decision qualification, refer to (16):

$$restriction(\vec{n},\vec{p},\vec{T}) = \begin{cases} \alpha\Pi_j(\vec{n},\vec{p},\vec{T}_i) \text{ if } ((\text{i}) \vee (\text{ii}) \vee (\text{iii})) \\ \Pi_j(\vec{n},\vec{p},\vec{T}) \text{ if} \neg ((\text{i}) \vee (\text{ii}) \vee (\text{iii})) \end{cases} \quad 0 \le \alpha < 1. \tag{16}$$

However, when dealing with the other approach for an appropriate decision (*i.e.* the one that best fits targets defined for the buyer and/or for the supplier), it is necessary to specify the strategic policies and then redefine only the evaluation process to guide the particles in direction of the targets pointed by that new policies. We assume that the policies can be divided in two sets: required or optional, and each strategic policy has a relative importance to the purchasing manager's decision. For both types of strategic policies (*i.e.* required or optional), to quantify each policy according to its importance, we define a set of weights ϖ. The weights are given as $\sum_{i=1}^{m_1}\varpi_{r_i} = 1$ and $\sum_{i=1}^{m_2}\varpi_{o_i} = 1$, where ϖ_{r_i} and ϖ_{o_i} are the i^th^ weight of the corresponding required and optional policies, respectively, and m_1 and m_2 are the respective quantities of strategic policies.

The new qualification of a decision $(\vec{n},\vec{p},\vec{T})$, based on those weights, is given as follows by (18).

$$new\_restriction(\vec{n},\vec{p},\vec{T}) = restriction(\vec{n},\vec{p},\vec{T}) + weighted\_\Pi(\vec{n},\vec{p},\vec{T}), \tag{18}$$

where,

$$weighted\_\Pi(\vec{n},\vec{p},\vec{T}) = (1-\alpha)\Pi_j(\vec{n},\vec{p},\vec{T}) \cdot \left(\sum_{i=1}^{m_2} 1_{\Pi_i}(\vec{n},\vec{p},\vec{T})\varpi_{o_i} - \sum_{i=1}^{m_1} 1_{\Pi_i}(\vec{n},\vec{p},\vec{T})\varpi_{r_i} \right), \tag{19}$$

and 1_{Π_i} is a step function as:

$$1_{\Pi_i}(\vec{n},\vec{p},\vec{T}) = \begin{cases} 1 \text{ if the ith condition is false for } (\vec{n},\vec{p},\vec{T}) \\ 0 \text{ if not} \end{cases}. \tag{20}$$

When PSO is applied to an optimization problem, each particle represents a possible solution to that problem. Hence, to optimize the Equation (18), we assume that each particle represents a decision $(\vec{n},\vec{p},\vec{T})$ in the problem domain.

The PSO algorithm applied to this inventory problem is given by Algorithm 1:

Algorithm-1:

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Step 0. Create, randomly, a set D containing N initial decisions (particles).
Step 1. Do while
Step 2.
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$$\left(\forall d_i \in D, \left(\left\|\vec{v}_i(t+1)\right\|^2 \neq 0 \vee \left\|\vec{P}_l(t) - \vec{x}(t+1)\right\|^2 \neq 0\right)\right)$$

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    (i) For each ith decision in D
        a) Evaluate the decision qualification using (18).
        b) Update P_l with max(P_l^{t+1}, P_l^t).
    (ii)Update P_g with max(P_g^{t+1}, P_g^t).
    (iii)Update the velocity and position of the particle according to (14) and (15), respectively.
Step 3. Return P_g as the best decision to this search cycle.
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The convergence condition to the PSO adopted in Algorithm 1 requires that: (i) the quadratic Euclidian norm of the particle's velocity vector to be zero; or, (ii) the quadratic Euclidian distance between the current particle's position and the local maximal or minimal position to be zero. This convergence condition was adapted from a previous work [15] to avoid negative numbers and additional wasted time within small values to the velocity and the relative position.

VI. Experiments

To validate the proposed approach to the inventory optimization problem, we compared the results obtained from our experimental simulations to those, selected randomly, from previous analytical studies [10]. Subsequently, some examples of strategic policies were presented and incorporated in the model. Therefore, Algorithm 1 is applied to that new optimization problem and a comparison between the suggested values for the model with and without policies were presented.

The objective of the second analytical study [10] was to analyze the influence of credit terms (*i.e.* M_{1_i} and M_{2_i}) on the supplier's, buyer's and the joint profit result. These results were obtained analytically to variations of M_{1_i} and M_{2_i} for only one product being retailed on only one storehouse.

All the parameter values of the inventory model used in the mentioned study were also adopted in this work to perform our first experimental simulations. The utilized parameters were: $a_1 = 25000$, $\rho_1 = 0.9$, $\delta_{11} = 1.25$, $c_1 = 2$, $v_1 = 4.5$, $S_{V_1} = 1000$, $S_{B_1} = 300$, $r_{V_1} = 0.05$, $r_{B_1} = 0.08$, $I_{Vp_1} = 0.09$, $I_{Bp_1} = 0.16$, $I_{Be_1} = 0.19$, $f_{Vc_1} = 0.17$ and $\beta_i = 0.02$. To the PSO algorithm, the numerical parameter values were: $\omega = 0.3$, $c_1 = 0.4$, $c_2 = 2.6$, $\alpha = 0.70$, and the number of particle was 35.

In Table I, there are results of six experiments. Each of them is composed for two rows of data: the first one consists of the analytical results [10] and the second, contains the experimental results, obtained by Algorithm 1. The optimal credit term for payment for each method is marked with "*".

TABLE I
COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

M_{1_1}	M_{2_1}	n_1	p_{11}	T_1 (days)	Profit Buyer	Profit Supplier	Profit Channel
10*	30	12	10.52	68.61	77987	31076	109063
*		13	10.52	67.44	77982	31081	109063
20*	30	13	10.57	66.03	78529	30471	109000
*		15	10.57	66.02	78529	30461	108990
0	60*	17	10.51	49.51	79276	31088	110364
	*	19	10.51	49.51	79276	31084	110360
10	60*	17	10.51	49.51	79276	31088	110364
	*	17	10.51	49.51	79276	31088	110364
0	90*	17	10.52	49.53	81370	30609	111979
	*	18	10.52	49.53	81376	30602	111978
10	90*	17	10.52	49.53	81370	30609	111979
	*	18	10.52	49.53	81370	30608	111978

Table I reveals that the experimental results are very close to the ones obtained analytically in the second numerical study [10], this for all experiments.

To illustrate the approach for determining an optimal decision according to strategic policies, we define a new scenario with an inventory composed of five identical products (*i.e.* the equivalent numerical values for all products are equal to each other). Each product P is retailed in two buyer's distinct storehouse (K_1 and K_2). Furthermore, the numerical values for the parameters of this inventory system were the same adopted in the first experimental study and, for all products, the credit terms were 20 and 90, in days. The objective of this study is to analyze the influence of the strategic policies over the values $\left(\vec{n}, \vec{p}, \vec{T}\right)$ suggested throughout the Algorithm 1.

For the second experimental study, we propose three strategic policies to be met by targets: (i) the products have to stay in stock during a time as small as possible (*i.e.* the value of T_i must be as small as possible); (ii) the retail price of the product P_3 in the storehouse K_1 has to be smaller than \$9.3, *i.e.* $p_{31} < 9.3$; (iii) it is desirable that the profit rate obtained by the retail of the product P_1 is 1.5 (*i.e.* the profit for P_1 divide by the cost for P_1, minus one). (i) and (ii) are required policies and their weights are $\varpi_{r_1} = 0.4$ and $\varpi_{r_2} = 0.6$, respectively. The third policy is optional and its weight is $\varpi_{o_1} = 1$.

The results for the second experimental study are presented in Table II. For this experiment we consider two situations: with or without the strategic policies. For each suggested variable, there are two rows. The first one contains the values generated by Algorithm 1 without policies. The second one contains the suggested values considering those defined strategic policies. For each situation, the experiment was repeated 30 times. Mean values and standard deviation are shown for each variable.

TABLE II
COMPARISON BETWEEN THE VALUES SUGGESTED BY ALGORITHM 1

	n_i		T_i		p_{i1}		p_{i2}	
P_1	17.01	0.01	0.0952	0	10.27	0.01	10.27	0.01
	16.31	2.66	0.1020	0	10.86	0.87	10.99	0.95
P_2	17.00	0.01	0.0673	0	10.23	0.00	10.24	0.01
	17.25	0.28	0.0677	0	10.38	0.80	10.25	0.07
P_3	17.03	0.23	0.0548	0	10.22	0.00	10.22	0.00
	16.94	0.61	0.0555	0	9.58	0.42	10.22	0.00
P_4	16.99	0.00	0.0475	0	10.20	0.03	10.21	0.00
	17.39	2.43	0.0476	0	10.21	0.00	10.21	0.00
P_5	16.28	2.84	0.0434	0	10.20	0.00	10.20	0.00
	16.90	0.48	0.0427	0	10.20	0.03	10.15	0.23

Notice that once the five products in stock were identical, we should expect that the generated values for each one be very close. The results for the algorithm with strategic policies are slightly different from those whose search process have not consider any policy. For the first required policy, the replenishment time had to be smaller than 0.05479 (*i.e.* 20 days). So, only for products P_1 and P_2, values of T_i have not satisfied that policy. To the second required policy, despite the retail price for P_3 in K_1 was greater than 9.3, $p_{31} < 10.22$.

The obtained mean profit rate for P_1 was 1.32 (standard deviation 0.12), value greater than those for all other products. The corresponding value without that optional policy was 1.23 (standard deviation of 0). Because lack of space these values are not in the Table II.

Table III shows the mean profit values of the experiments for both situations presented above. The results are presented in a pair of rows (*i.e.* without and with policy, respectively).

TABLE III
COMPARISON BETWEEN THE PROFIT VALUES ACHIEVED BY ALGORITHM 1

	Expected Profit	Standard Deviation	Profit Gain %
Buyer	2485218.75	543.92	0.00
	2497592.77	26861.19	0.50
Supplier	970338.20	1008.04	0.00
	955744.50	29089.06	-1.50
Channel	3455556.96	930.20	0.00
	3453337.27	2443.92	-0.06

It is important to mention that the total expected profit (*i.e.* buyer, supplier and joint) generated for those inventory models with and without the policies were very close to each other.

VII. CONCLUSION

In this article, we extended the original integrated supplier-buyer inventory model [10] with multi-storehouse and multi-product. We also developed a mechanism to incorporate strategic policies in that new inventory model. In order to find an optimal solution to that new model, we applied an approach based on PSO. To validate that approach, a comparison between the experimental results and the analytical ones was carried out.

The first experimental simulations (refer to Table I) reveled that Algorithm-1 is a suitable alternative for the original optimization analytical process [10]; needless to stress the advantage of having this process performed automatically.

The second experimental simulations revealed (refer to Table II) that the mechanism of strategic policies is a useful way to customize the new integrated inventory model to each company's needs.

The last simulations revealed (refer to Table III) that by using policies, although slightly, profit indeed can increase.

We conclude by pointing out that in the future, our approach can be improved if a probabilistic demand function is used instead of a deterministic one. Multi-objective optimization techniques are also an avenue to pursue as an alternative for the weighted sums of strategies (objectives).

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