

# On the Influence of the Swimming Operators in the Fish School Search Algorithm

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**Abstract**—Real world engineer problems sometimes involve high dimensional spaces. It makes them hard to compute. A common approach to tackle such challenges is to apply swarm based or evolutionary algorithms. Fish School Search (FSS) is one of such techniques that excels on difficult search problems. As FSS is a recent technique and only output results were investigated so far. This paper analyzes the influence of the FSS operators on the performance of the algorithm in six benchmark functions. We assessed the influence of each swimming operator separately. We found that the volitive mechanism is the operator that affords most of the exploration abilities during the search process. The carried out assessment has also shown that, in average, the best results are obtained only when all the FSS operators are activated. It means that all operators are fairly relevant and complementary. Moreover, we compared FSS results with some PSO variations and showed that FSS outperformed these PSO algorithms in some cases.

**Index Terms**—Fish School Search, Optimization, Swarm Intelligence.

## I. INTRODUCTION

Swarm intelligence has been frequently used to tackle optimization problems in high dimensional continuous search spaces. Many approaches were proposed in recent years, such as Particle Swarm Optimization (PSO) [1] [2] [3], Ant Colony Optimization (ACO) [4], Artificial Bee Colony Algorithm (ABC) [5], among others.

Fish School Search (FSS) [6] is a novel approach recently proposed to perform search in multimodal problems. Some preliminary simulations indicated that FSS can outperform many bio-inspired algorithms, mainly in multimodal functions.

FSS has a set of operators inspired on swimming and feeding. This paper analyzes the influence of each FSS swimming operator in a set of benchmark functions [7]. This analysis aims to verify the real influence of each FSS swimming operator during the search process.

This paper is organized as follows: in section II we give an overview of the FSS algorithm. In section III we detail the simulation setup, including references to all benchmark functions and FSS parameters. In section IV simulation results are presented. We performed four distinct analysis. The first two are concerned with the influence of each individual operator. The third one analyses the FSS dependence on its parameters. And the last one compares FSS to other approaches of PSO in terms of performance and execution time. In section V we discuss the results and give our conclusions.

## II. FISH SCHOOL SEARCH ALGORITHM

The FSS algorithm was inspired in the gregarious behavior presented by many pelagic fish species. This social phenomenon can be understood at least in two different ways: mutual protection and synergy to perform collective tasks, both to improve survivability of the whole group. Mutual protection means reducing the chances of being chased and caught by predators (i.e. it can be used to avoid local minima). On the other hand, synergy to perform collective tasks refers to the communication capacity among the fish to achieve collective goals such as finding food (i.e. it can be used to increase the success rate of the search) [6].

In nature, some fish species live their entire lives in schools; despite this reduces the individual freedom of swimming, it increases competition in scarce food regions. Thus, the benefits largely outweighs the drawbacks. The main characteristics were derived from real fish schools and incorporated into the core of our approach.

The search process in FSS is carried out by a population of bounded-memory individuals, the fish. Each fish represents a possible solution to the problem. In this regard, they are similar to particles in the PSO algorithms and individuals in Genetic Algorithms.

To understand the FSS algorithm, a number of concepts need to be defined. The aquarium is the region in the search space where the fish can be positioned and are allowed to move. Food density in the aquarium, for example, is related to the function to be optimized in the search space. That is, in a minimization problem the amount of food in a particular region is inversely proportional to the function evaluation in this region.

The algorithm starts with all fish at random positions. The current version of the FSS algorithm has four operators, which can be grouped in two classes: feeding and swimming. Feeding represents the quality of a solution for the problem and the swimming drives the fish movements. Search guidance in FSS is only driven by successful fish. FSS has three swimming operators to move the fish according to the feeding operator. In the next subsections, we present in details the four FSS operators.

### A. Individual movement operator

The individual movement operator is applied in each iteration for each fish in the school. Each fish randomly chooses

a new position in its neighborhood and evaluates it by using the fitness function.

The next candidate position is determined by adding to each dimension of the current position a random number generated by a uniform distribution in the interval [-1,1] multiplied by a predetermined step ( $step_{ind}$ ) as shown in (1).

$$n_i(t) = x_i(t) + rand(-1, 1) step_{ind}, \quad (1)$$

where  $x_i$  is the current position of the fish in dimension  $i$ ,  $n_i$  is the neighbor position of the fish in dimension  $i$ ,  $rand()$  is a function which returns a random number from a uniform distribution in a specified interval.

Despite one candidate new position is generated in each iteration for each fish, the movement only occurs if the new position has a better fitness than the current one. The fitness difference ( $\Delta f$ ) and the displacement ( $\Delta \vec{x}$ ) are evaluated according to (2) and (3), respectively. One should notice that  $\Delta \vec{x}$  is only evaluated if the individual movement occurs. Otherwise,  $\Delta \vec{x} = 0$

$$\Delta f = f(\vec{n}) - f(\vec{x}), \quad (2)$$

$$\Delta \vec{x} = \vec{n} - \vec{x}. \quad (3)$$

The  $step_{ind}$  is a percentage of the search space amplitude and decreases linearly during iterations by using (4) in order to improve the exploitation ability in later iterations.

$$step_{ind}(t+1) = step_{ind}(t) - \frac{(step_{ind\ initial} - step_{ind\ final})}{iterations}, \quad (4)$$

where  $iterations$  is the number of iterations used in the simulation. The  $step_{ind\ initial}$  and  $step_{ind\ final}$  are the initial and the final individual movement step, respectively. Obviously, the  $step_{ind\ initial}$  must be higher than  $step_{ind\ final}$ .

### B. The feeding operator

All fish are born with the same weight and start with weight equal to 1. Each fish can increase its weight depending on the success rate achieved by the individual movement. The fish weight variation is proportional to the normalized difference between the evaluation of the fitness function at current and new position. The weight is updated according to (5).

$$W_i(t+1) = W_i(t) + \frac{\Delta f_i}{max(\Delta f)}, \quad (5)$$

where  $W_i(t)$  is the weight of the fish  $i$ , ( $\Delta f_i$ ) is the difference of the fitness at current and new position for the fish  $i$ ,  $max(\Delta f)$  is a function that returns the maximum difference of the fitness values among all the fish. One should remember that  $\Delta f_i = 0$  for fish that did not perform the individual movement at the current iteration.

### C. Collective instinctive movement operator

Only fish that successfully performed individual movements, i.e.  $\Delta \vec{x}_i \neq 0$ , influence the resulting direction of the school movement. The resulting direction ( $\vec{I}$ ) is evaluated by using

(6). After that, all fish of the school must update their positions according to (7).

$$\vec{I}(t) = \frac{\sum_{i=1}^N \Delta \vec{x}_i \Delta f_i}{\sum_{i=1}^N \Delta f_i}, \quad (6)$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{I}(t). \quad (7)$$

### D. Collective volitive movement operator

This movement is based on the overall success rate of the whole fish school. If the fish school weight is increasing, it means that the search has been successful. Then, the radius of the school should contract to increase the exploitation ability. Otherwise, it should increase.

The fish-school contraction is applied as a step inward drift to every fish position with regard to the school barycenter.

The fish-school barycenter is obtained by considering all fish positions and their weights, as shown in (8). All fish must update their positions according to (9) when the total weight of the school increased at the current iteration, or according to (10) when the total weight of the school decreased at the current iteration.

$$\vec{B}(t) = \frac{\sum_{i=1}^N \vec{x}_i w_i(t)}{\sum_{i=1}^N w_i(t)}, \quad (8)$$

$$\vec{x}(t+1) = \vec{x}(t) - step_{vol} rand(0, 1) \frac{(\vec{x}(t) - \vec{B}(t))}{distance(\vec{x}(t), \vec{B}(t))}, \quad (9)$$

$$\vec{x}(t+1) = \vec{x}(t) + step_{vol} rand(0, 1) \frac{(\vec{x}(t) - \vec{B}(t))}{distance(\vec{x}(t), \vec{B}(t))}, \quad (10)$$

where  $distance()$  is a function which returns the Euclidean distance between the barycenter and the fish current position,  $step_{vol}$  is a predetermined step used to control the displacement from/to the barycenter.

The  $step_{vol}$  must be in the same order of magnitude of the step used in the individual movement. As  $step_{vol}$  is multiplied by a uniform distribution in interval [0,1] with expected value equal to 0.5, we propose the  $step_{vol}$  value to be twice the  $step_{ind}$  value.

### E. FSS Pseudo code

The FSS implementation for minimization problems follows the pseudo code shown in Algorithm 1.

## III. EXPERIMENTAL SETUP

The following benchmarks functions are minimizing problems and were used in our experiments.

$$F_{Rosenbrock}(\vec{x}) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2], \quad (11)$$

$$F_{Rastrigin}(\vec{x}) = 10n + \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i)], \quad (12)$$

$$F_{Griewank}(\vec{x}) = 1 + \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right), \quad (13)$$

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**Algorithm 1** FSS Pseudo code

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```
initialize randomly all fish
while stop criterion is not met do
  for each fish do
    evaluate fitness function
    individual movement by using (1)(2)(3)
    feeding by using (5)
    evaluate the fitness function
  end for
  for each fish do
    instinctive movement by using (6)(7)
  end for
  Calculate barycentre using (8)
  for each fish do
    volitive movement by using (9) or (10)
  end for
  update step using (4)
end while
```

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$$F_{Ackley}(\vec{x}) = -20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left( \frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + e, \quad (14)$$

$$F_{Shwefel1.2}(\vec{x}) = \sum_{i=1}^n \left( \sum_{j=1}^i x_j \right)^2, \quad (15)$$

$$F_{Sphere}(\vec{x}) = \sum_{i=1}^n x_i^2. \quad (16)$$

The search space, the initialization subspace and optimum point in the search space for the benchmark functions are shown in Table I. Notice that the optimum point for all the benchmark functions is always placed far away from the initialization subspace. We did this to test the exploration ability of our algorithm.

TABLE I

THE SEARCH SPACE, THE INITIALIZATION SUBSPACE AND THE OPTIMUM POINT IN THE SEARCH SPACE FOR THE BENCHMARK FUNCTIONS USED IN THE SIMULATIONS.

Function	Search space	Initialization subspace	Optimum point
Rosenbrock	$-30 \leq x_i \leq 30$	$15 \leq x_i \leq 30$	$1.0^D$
Rastrigin	$-5.12 \leq x_i \leq 5.12$	$2.56 \leq x_i \leq 5.12$	$0.0^D$
Griewank	$-600 \leq x_i \leq 600$	$300 \leq x_i \leq 600$	$0.0^D$
Ackley	$-32 \leq x_i \leq 32$	$16 \leq x_i \leq 32$	$0.0^D$
Schwefel 1.2	$100 \leq x_i \leq 100$	$50 \leq x_i \leq 100$	$0.0^D$
Sphere	$100 \leq x_i \leq 100$	$50 \leq x_i \leq 100$	$0.0^D$

In all the simulations, we performed 30 trials per function, where 5,000 iterations were executed for each trial in a 30 dimensions search space. We used 30 fishes in all the simulations. The total number of fitness functions evaluations

was 300,000. One should notice that each fish has to evaluate the fitness twice per iteration as the fish must check the fitness before and after its individual movement.

## IV. RESULTS

The experiments carried out in this paper included various simulations schema of switched on and off operator and some different sets of parameter values.

Four configurations of operators were tested in this paper to evaluate the effectiveness of the operators themselves, they were: (i) only individual movement; (ii) individual movement, feeding and instinctive collective movement; (iii) individual movement, feeding and volitive collective movement; (iv) all the operators.

## A. Operators analysis

Because of the formulation of the FSS operators, a precedence order had to be established. First, we switched-on only the individual operator, then we switched-on three different operators simultaneously with the individual movement: instinctive, volitive, and instinctive plus volitive movements. Note that the feeding operator has to be switched-on when using instinctive or volitive movements. The set of experiments, used three ranges for  $step_{ind\ initial}$  and  $step_{ind\ final}$ : 10% to 0.01%, 1% to 0.001% and 10% to 0.001%. One must remember that  $step_{vol}$  is equal to twice times  $step_{ind}$ .

Table II, Table III and Table IV present the comparative performance of the FSS algorithm on each benchmark function with operators selectively switched on and off.

By using  $step_{ind}$  linearly decreasing from 10% to 0.01% (see table II), one can observe that best results are obtained when all the operators are switched on. This behavior occurred for all benchmarks functions except Rosenbrock.

By using  $step_{ind}$  linearly decreasing from 1% to 0.001% (see table III), one can observe that for multimodal functions (Ackley and Rastrigin), the FSS algorithm apparently got stucked in local minima. It might be because  $step_{ind}$  is too small to provide good exploration abilities in the search space. For Rosenbrock function, a simple local greedy search performed better than a more comprehensive approaches with less exploration.

By using  $step_{ind}$  linearly decreasing from 10% to 0.001% (see table IV), one can observe the same behavior presented in table II.

## B. Graphical analysis

We developed a tool to analyse the influence of each operator during the search process. This tool counts how many times a fish improved its positions and how many times the fish school improved the overall current result of the search.

Figures 1, 2, 3, 4, 5 and 6 present the fitness, the number of times a fish improved its own position during the simulations and the number of times that the fish school improved the overall result during the simulations for all the six benchmark functions. The figures show the individual, instinctive and volitive operators contribution to the search task along all iterations.

TABLE II

MEAN VALUE AND STANDARD DEVIATION FOR 30 TRIALS OF 300,000 FITNESS EVALUATIONS USING  $step_{ind}$  DECREASING FROM 10% TO 0.01%

Function	individual	individual, feeding, instinctive	individual, feeding, volitive	individual, feeding, instinctive, volitive
Rosenbrock	28.7383 (0.9624)	61.6364 (36.6475)	222.5643 (133.4478)	30.688 (16.4725)
Rastrigin	281.0138 (23.5627)	89.6149 (10.8363)	187.6666 (13.6402)	72.6011 (15.6294)
Griewank	0.2833 (0.0476)	0.6927 (0.0756)	0.1727 (0.0318)	0.0248 (0.0097)
Ackley	19.7416 (0.0395)	1.3039 (0.3163)	0.2042 (0.1717)	0.0165 (0.0025)
Schwefel 1.2	21.0597 (4.4788)	400.8578 (80.0968)	10.2946 (4.5010)	1.2443 (0.4890)
Sphere	0.0971 (0.0165)	0.6368 (0.1473)	0.0239 (0.0052)	0.0044 (0.0013)

TABLE III

MEAN VALUE AND STANDARD DEVIATION FOR 30 TRIALS OF 300,000 FITNESS EVALUATIONS USING  $step_{ind}$  DECREASING FROM 1% TO 0.001%

Function	individual	individual, feeding, instinctive	individual, feeding, volitive	individual, feeding, instinctive, volitive
Rosenbrock	7.399 (2.7243)	58.868 (35.4825)	16.6632 (1.1757)	101.3338 (11.7628)
Rastrigin	386.3746 (13.5628)	385.9176 (15.9174)	460.3525 (11.3811)	437.4146 (15.5405)
Griewank	0.0061 (0.0015)	0.0416 (0.0091)	0.0073 (0.0034)	0.0054 (0.0057)
Ackley	19.7708 (0.0160)	19.787 (0.0371)	19.819 (0.0061)	19.8075 (0.0227)
Schwefel 1.2	0.337 (0.0775)	6.0045 (1.5089)	0.312 (0.1185)	0.0144 (0.0058)
Sphere	0.0008 (0.0002)	0.0065 (0.0012)	0.0002 (0.0000)	0.0000 (0.0000)

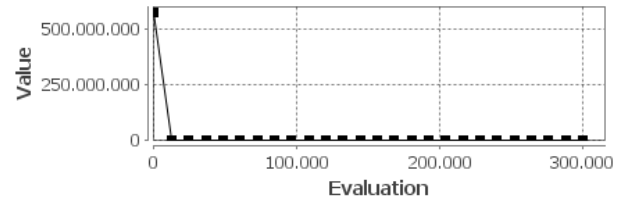
TABLE IV

MEAN VALUE AND STANDARD DEVIATION FOR 30 TRIALS OF 300,000 FITNESS EVALUATIONS USING  $step_{ind}$  DECREASING FROM 10% TO 0.001%

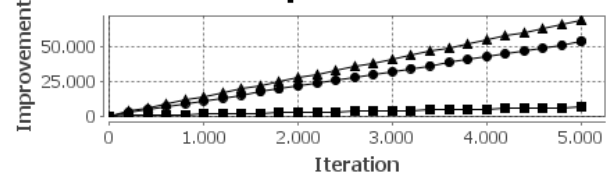
Function	individual	individual, feeding, instinctive	individual, feeding, volitive	individual, feeding, instinctive, volitive
Rosenbrock	37.6834 (5.6466)	53.5716 (15.4586)	194.3466 (94.9955)	31.6209 (16.1681)
Rastrigin	298.0803 (26.0647)	88.6092 (11.8655)	164.0563 (20.2838)	83.2125 (20.3606)
Griewank	0.5405 (0.0528)	0.7249 (0.0580)	0.1363 (0.0346)	0.0211 (0.0079)
Ackley	19.8316 (0.0328)	1.2549 (0.2844)	0.2108 (0.2047)	0.0108 (0.0021)
Schwefel 1.2	96.5609 (22.0749)	407.9883 (74.1360)	8.1067 (3.4033)	1.2793 (0.4490)
Sphere	0.3317 (0.0707)	0.6076 (0.1049)	0.0203 (0.0042)	0.0019 (0.0005)

The fitness results for all the benchmark functions confirm that the algorithm is quite effective in a broad range of complex problems. From the other graphs, one can note that the volitive and instinctive operators, in this order, are the operators responsible by the greatest number of improvements

Best Fitness

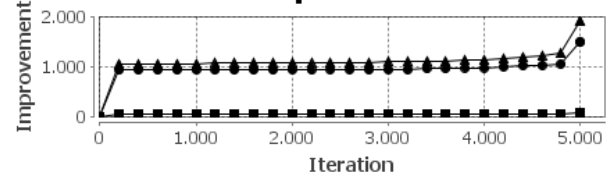


Local Improvements



■ Individual ● Instinctive ▲ Volitive

Global Improvements



■ Individual ● Instinctive ▲ Volitive

Fig. 1. Simulation results for Rosenbrock function: fitness, number of fish fitness improvements along the search and number of overall fitness improvements as a function of the number of the iterations.

during the search process. However, the individual operator cannot be disregarded at all, because it is responsible for triggering all the other swimming operators.

### C. Parameter analysis

After assessing each operator, we investigated the influence of the step size on the FSS performance. Table V shows the influence of the  $step_{ind\ final}$  in the search process for  $step_{ind\ initial} = 10\%$ . One can notice that the best results were achieved for  $step_{ind\ final}$  equal to 0.001% and 0.0001%.

TABLE V

MEAN VALUE AND STANDARD DEVIATION FOR 30 TRIALS OF 300,000 FITNESS EVALUATIONS WITH ALL OPERATORS SWITCHED-ON

Function	step 10% to 0.01%	step 10% to 0.001%	step 10% to 0.0001%	step 10% to 0.00001%
Rosenbrock	29.0679 (0.1946)	27.8284 (0.8989)	26.9713 (0.9236)	27.5188 (1.3834)
Rastrigin	79.2671 (16.9310)	79.2658 (1.6911)	74.6226 (11.7165)	89.5468 (8.0010)
Griewank	0.0195 (0.0070)	0.0153 (0.0046)	0.0323 (0.0081)	0.024 (0.0065)
Ackley	0.0177 (0.0015)	0.0106 (0.0019)	0.0110 (0.0029)	0.0089 (0.0009)
Schwefel 1.2	1.1825 (0.2299)	1.1664 (0.2996)	1.3672 (0.1409)	1.2691 (0.3901)
Sphere	0.0042 (0.0008)	0.0024 (0.0009)	0.0024 (0.0010)	0.0017 (0.0003)

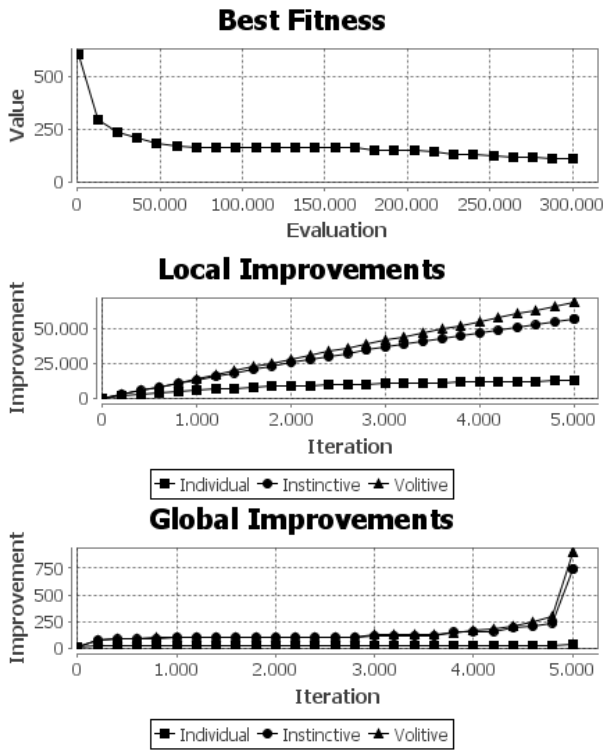


Fig. 2. Simulation results for Rastrigin function: fitness, number of fish fitness improvements along the search and number of overall fitness improvements as a function of the number of the iterations.

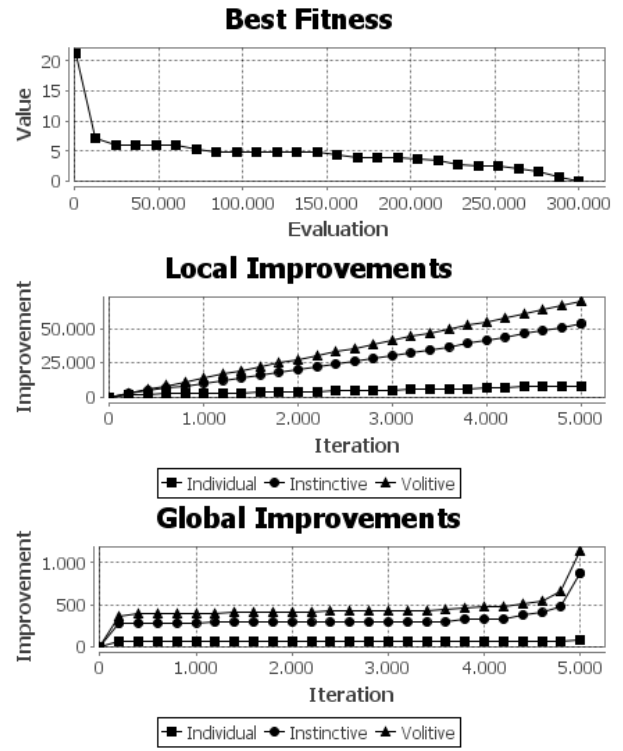


Fig. 4. Simulation results for Ackley function: fitness, number of fish fitness improvements along the search and number of overall fitness improvements as a function of the number of the iterations.

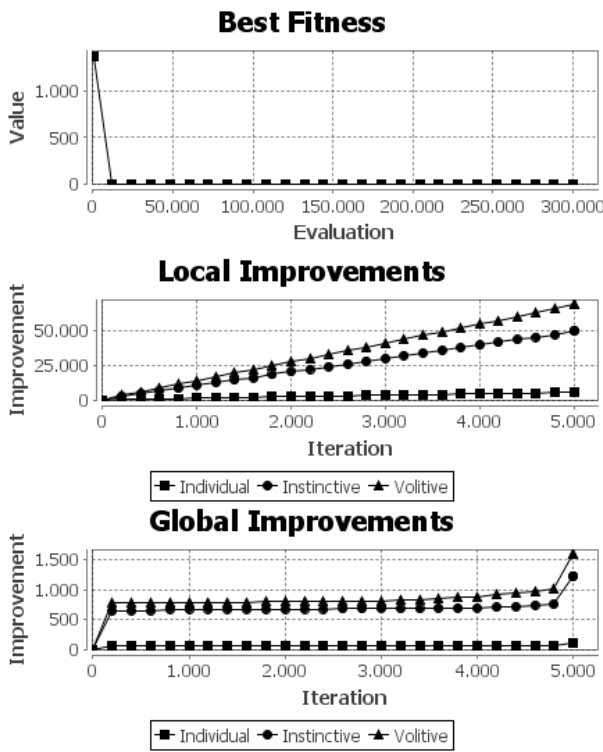


Fig. 3. Simulation results for Griewank function: fitness, number of fish fitness improvements along the search and overall fitness improvements as a function of the number of the iterations.

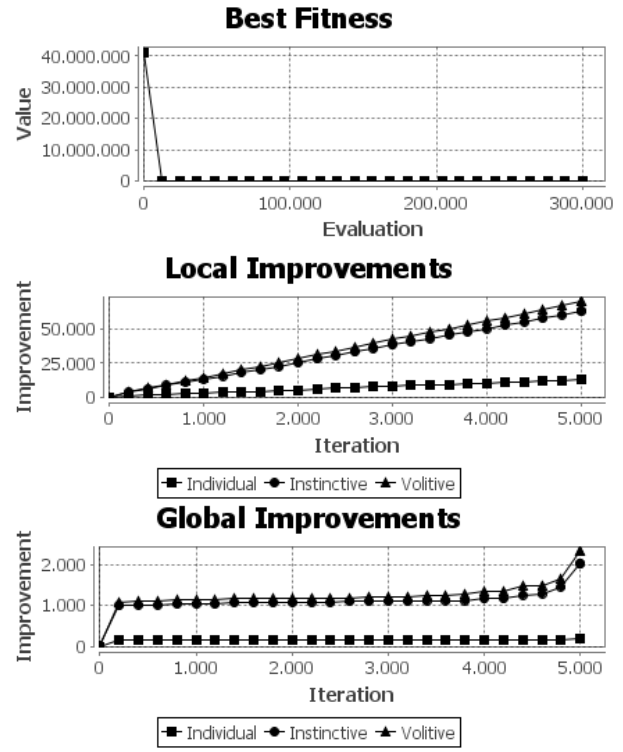


Fig. 5. Simulation results for Schwefel 1.2 function: fitness, number of fish fitness improvements along the search and number of overall fitness improvements as a function of the number of the iterations.

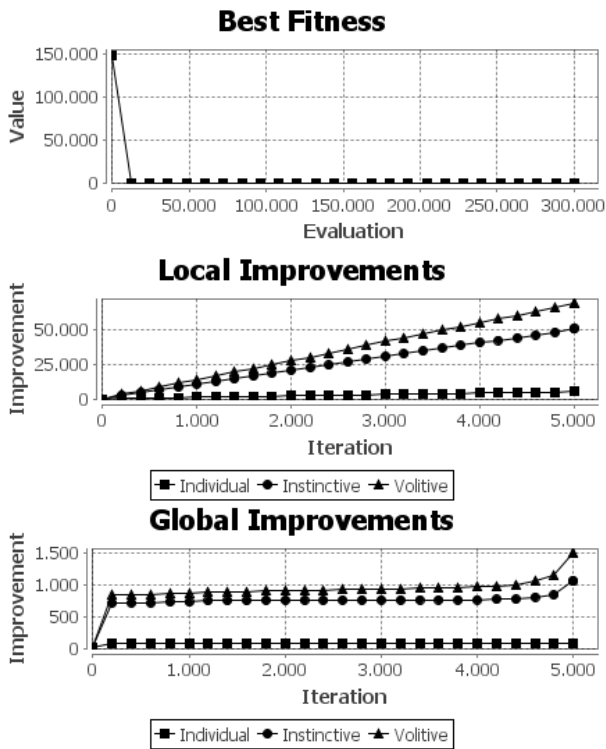


Fig. 6. Simulation results for Sphere function: fitness, number of fish fitness improvements along the search and number of overall fitness improvements as a function of the number of the iterations.

TABLE VI  
MEAN VALUE AND STANDARD DEVIATION FOR COMPARATIVE STUDY

Function	Original PSO	Constricted PSO Global Best	Constricted PSO Local Best	FSS step 10% to 0.0001%
Rosenbrock	54.6867 (2.8570)	8.1579 (2.7835)	12.6648 (1.2304)	26.9713 (0.9236)
Rastrigin	400.7194 (4.2981)	140.4876 (4.8538)	144.8155 (4.4066)	74.6226 (11.7165)
Griewank	1.0111 (0.0031)	0.0308 (0.0063)	0.0009 (0.0005)	0.0323 (0.0081)
Ackley	20.2769 (0.0082)	17.6628 (1.0232)	17.5891 (1.0264)	0.0110 (0.0029)
Schefel 1.2	5.4572 (0.1429)	0.0000 (0.0000)	0.1259 (0.0178)	1.3672 (0.1409)

#### D. Performance Comparison with PSO

In Table VI, we compare FSS and three PSO [1] [2] [3] variations. The results for PSO were extracted from [1]. As can be seen, the FSS achieved better results in some multimodal functions such as Rastrigin and Ackley. One can also note the FSS obtained a similar performance to PSO using global topology in the Griewank function.

Table VII presents the mean value and standard deviation for the execution time to perform FSS and PSO for 30 trials of 300,000 fitness evaluations. As can be seen, the FSS achieved better results in almost all functions. The results are even better in FSS when the algorithms are evaluating simple functions, for example, Sphere. It happens because, as

FSS evaluates twice the objective function in each iteration, then FSS performs a half number of iterations than PSO to accomplish the same number of fitness evaluations.

TABLE VII  
MEAN VALUE AND STANDARD DEVIATION OF EXECUTION TIME FOR 30 TRIALS OF 300,000 FITNESS EVALUATIONS

Function	FSS	PSO
Ackley	4.4891 (0.1448)	4.3172 (0.2264)
Sphere	1.5104 (0.0116)	3.4682 (0.0131)
Griewank	5.0849 (0.0147)	4.6495 (0.2840)
Rastrigin	4.4724 (0.1192)	4.7542 (0.1863)
Rosenbrock	1.7677 (0.0104)	3.5234 (0.0244)
Schwefel	2.6958 (0.0119)	3.8099 (0.0114)

#### V. DISCUSSION AND CONCLUSIONS

This paper investigated the importance of FSS operators on the performance of the FSS algorithm on well-known benchmark functions. We observed that all operators exert strong influence in the search result. A simple proof can be obtained by removing any operator of current version of the FSS algorithm as can be seen in Figures 1-6.

Sometimes the individual operator, alone, produces better results than all operator put together. This is a counter intuitive result explained by the high variability of some functions.

Although it is not an initial objective of this paper, we showed that good results of FSS are always yield by starting up the algorithm on large exploration mode (i.e.  $step_{ind} = 10\%$ ).

In such cases, the algorithm should incorporate an automatic mechanism to disable temporarily some operators. As it is, FSS is slightly naive on that regard, although it already performs quite well even for difficult functions such as Rosenbrock.

Finally, another important future task is to assess the impact of the breeding operator (i.e. the fifth FSS operator - not used here).

#### ACKNOWLEDGMENT

The authors thank the Polytechnic School of Pernambuco, FACEPE and CNPq for the financial support for this paper.

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